Lesson 27
Solve Problems with Cylinders, Cones, and Spheres

Prerequisite: Find the Volume of Cones and Cylinders

Study the example problem showing how to find the volume of a cylinder and a cone. Then solve problems 1–7.

Example
Find the volumes of the cone and the cylinder. Write the volumes in terms of $\pi$.

<table>
<thead>
<tr>
<th>Volume of Cone</th>
<th>Volume of Cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V = \frac{1}{3} Bh$</td>
<td>$V = Bh$</td>
</tr>
<tr>
<td>$= \frac{1}{3} \pi r^2 h$</td>
<td>$= \pi r^2 h$</td>
</tr>
<tr>
<td>$= \frac{1}{3} \pi \cdot 6^2 \cdot 5$</td>
<td>$= \pi \cdot 6^2 \cdot 5$</td>
</tr>
<tr>
<td>$= 60\pi$</td>
<td>$= 180\pi$</td>
</tr>
</tbody>
</table>

The volume of the cone is $60\pi$ cubic centimeters, and the volume of the cylinder is $180\pi$ cubic centimeters.

1. In the example, what measurements are the same in the cone and the cylinder?

2. How does the volume of the cone in the example compare to the volume of the cylinder?

3. Suppose the height of the cone in the example was tripled. How would the volume of the cone compare to the volume of the cylinder? Explain how you know.

Vocabulary

- **cylinder** a solid figure with two congruent and parallel circular bases.
- **cone** a solid figure with one vertex and one circular base.
Solve.

Use the figures for problems 4–6.

Volume of cylinder $= \pi r^2 h \quad$ Volume of cone $= \frac{1}{3}\pi r^2 h$

4. Find the volume of each solid figure. Write your answers in terms of $\pi$, $r$, and $h$.

   A: _______  C: _______
   B: _______  D: _______

5. Order the solid figures from least volume to greatest volume. Explain your reasoning.

6. Sheila says that a cone with a radius of $2r$ and a height of $6h$ would have the same volume as one of the solid figures above. Do you agree with Sheila? Explain why or why not.

7. A cylinder and a cone have the same radius. The volume of the cone is twice the volume of the cylinder. How many times greater is the height of the cone than the height of the cylinder? Explain your reasoning.
Solve Volume Problems

Study the example problem showing how to solve a volume problem. Then solve problems 1–8.

Example

Two solid glass paperweights are shown at the right. What is the volume of glass used to make each paperweight? Write the volumes in terms of $\pi$.

<table>
<thead>
<tr>
<th>Volume of Cone</th>
<th>Volume of Cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V = \frac{1}{3} Bh$</td>
<td>$V = Bh$</td>
</tr>
<tr>
<td>$= \frac{1}{3} \pi r^2 h$</td>
<td>$= \pi r^2 h$</td>
</tr>
<tr>
<td>$= \frac{1}{3} \pi \cdot 4^2 \cdot 6$</td>
<td>$= \pi \cdot 4^2 \cdot 3$</td>
</tr>
<tr>
<td>$= 32\pi$</td>
<td>$= 48\pi$</td>
</tr>
</tbody>
</table>

The volume of glass used in the cone paperweight is $32\pi$ cubic centimeters, and the volume of glass used in the cylinder paperweight is $48\pi$ cubic centimeters.

1. In the example, was more glass used to make the cone paperweight or the cylinder paperweight? How much more?

2. In the example, why was 4 and not 8 substituted for $r$ to find the volume of the cylinder?

3. The paperweight manufacturer decides to change the height of the cone paperweight in the example so that the same volume of glass is used to make both paperweights. What will be the new height of the cone paperweight? Explain your reasoning.
Solve.

Volume of cylinder = $\pi r^2 h$  Volume of cone = $\frac{1}{3} \pi r^2 h$

4 Find the volume of the cylinder-shaped grain storage tank at the right. Write the volume in terms of $\pi$.

5 Find the volume of the cone-shaped grain storage tank at the right. Write the volume in terms of $\pi$.

6 For the tanks in problems 4 and 5, calculate each volume using 3.14 for $\pi$.

Cylinder-shaped tank: ________________________________

Cone-shaped tank: ________________________________

7 Jacob is buying cups for his frozen yogurt business. He can choose from the cups shown at the right. He wants to buy the type that will hold more frozen yogurt.

a. Find the volume for each type of cup. Use 3.14 for $\pi$, and round to the nearest tenth.

____________________________________________________

____________________________________________________

b. Which type of cup should Jacob buy? Explain.

____________________________________________________

____________________________________________________

8 The height of a cylinder is 6 centimeters and the volume is 150$\pi$ cubic centimeters. A cone has the same volume as the cylinder. Give two possible radius and height combinations for the cone: one with the same radius as the cylinder, and one with a different radius. Explain your reasoning.

____________________________________________________

____________________________________________________

____________________________________________________

____________________________________________________
Compare Volumes

Study the example showing how to solve problems involving volume. Then solve problems 1–6.

Example

An artist is making three solid figures out of clay. The designs for the solid figures are shown at the right. Find the volume of all three figures.

Use 3.14 for \( \pi \), and round the answers to the nearest hundredth. Which figure requires the most clay?

<table>
<thead>
<tr>
<th>Sphere</th>
<th>Cylinder</th>
<th>Cone</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ V = \frac{4}{3} \pi r^3 ]</td>
<td>[ V = Bh ]</td>
<td>[ V = \frac{1}{3} Bh ]</td>
</tr>
<tr>
<td>[ = \frac{4}{3} \pi (2)^3 ]</td>
<td>[ = \pi r^2 h ]</td>
<td>[ = \frac{1}{3} \pi r^2 h ]</td>
</tr>
<tr>
<td>[ = \frac{4}{3} (3.14)(2)^3 ]</td>
<td>[ = 3.14 \cdot (1.5)^2 \cdot 4 ]</td>
<td>[ = \frac{1}{3} (3.14)(3)^2(3) ]</td>
</tr>
<tr>
<td>[ \approx 33.49 \text{ in.}^3 ]</td>
<td>[ = 28.26 \text{ in.}^3 ]</td>
<td>[ = 28.26 \text{ in.}^3 ]</td>
</tr>
</tbody>
</table>

The volume of the sphere is the greatest and therefore requires the most clay.

1. How much clay does the artist need to make all three figures?

2. If the artist begins with a spherical ball of clay with a radius of 3 inches, does she have enough clay to make all three figures? Explain.

3. Does the artist have enough clay to make three of the spheres shown instead of one of each shape? Explain.
Solve.

Volume of cylinder = \( \pi r^2 h \)  Volume of cone = \( \frac{1}{3} \pi r^2 h \)  Volume of sphere = \( \frac{4}{3} \pi r^3 \)

4 Two bowls are half-spheres. Find the volumes of the bowls. Use 3.14 for \( \pi \), and round the answers to the nearest whole number. How many times greater is the volume of the large bowl than the volume of the small bowl?

Show your work.

Solution: 

Use the given information and the figures shown to solve problems 5–6.

A toy company makes sphere-shaped and cone-shaped toys. Two of the toys are shown at the right.

5 Lawrence says that because the height and radius of the cone are equal to the radius of the sphere, the two toys have the same volume. Explain why Lawrence is incorrect.

6 Change one of the dimensions of the cone-shaped toy so that the volumes of the toys are the same.
The propane storage tank shown is a cylinder with a half-sphere on each end.

Tell whether each statement is True or False.

a. The volume of the cylinder part of the tank is about 550 ft³.
   - True
   - False

b. The volume of one of the half-spheres is about 65 ft³.
   - True
   - False

c. The combined volume of the two half-sphere parts is less than the volume of the cylinder part.
   - True
   - False

d. The volume of the tank is about 202 ft³.
   - True
   - False

Tom is making strawberry jelly and is going to put it into the jar shown. About how much jelly will he need to fill the jar to 0.5 inch from the top? Circle the correct answer. (Use 3.14 for π, and round to the nearest whole.)

A 28 in.³
B 57 in.³
C 63 in.³
D 127 in.³

Isabelle chose C as the correct answer. How did she get that answer?
Solve.

3 Juan cut a section shaped like a cone out of the center of a piece of wood that is shaped like a cylinder, as shown below. What is the volume of the piece of wood left after Juan cut out the cone-shaped section? Use 3.14 for \( \pi \), and round to the nearest whole cubic centimeter.

Show your work.

Solution: ___________________________________________________________________________

4 A barrel in the shape of a cylinder is cut in half lengthwise to make a water trough for horses. Does the expression \( \pi \left( \frac{1}{2}r \right)^2 h \) represent the volume of water that the water trough holds? Explain why or why not.

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

5 The barrel in problem 4 is 5 feet long and the radius of its base is 1.5 feet. How much water will the water trough hold? Use 3.14 for \( \pi \), and round to the nearest whole cubic foot.

___________________________________________________________________________
___________________________________________________________________________

Don’t forget to round your answer.